SPATIOTEMPORAL CHAOS OF SOLITON IN A GENERALIZED SKYRME MODEL WITH THE MODIFIED SYMMETRY - BREAKING TERM .

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Abstract

The chiral symmetry-breaking term of the Skyrme model with massive pion is modified to obtain the hedgehog profile function which is in best coincidence with the kink-like profile function. For the modified Lagrangian, the minimum of the energy of the B=2 twisty skyrmion configuration is lower than the values for both the cases of the Skyrme Lagrangian with and without the non-modified symmetry-breaking term. The equations of motion for the time-dependent hedgehog of this model and for a generalizated Skyrme model including sixth-order stabilizing term are derived and integrated nummerically. The time evolution of soliton is obtained. We have observed the seft-exitation of soliton because of the fast developement of fluctuation.

I. INTRODUCTION

It is known that one can give a description of low energy hadron physics on the base of a semiclassical quatization of the soliton solution of the Skyrme model (skyrmion) [1-3]. The skyrmion configuration, denoted by $U(\overrightarrow{x})$, is a map: $R^3 \to SU(2)$, with the condition $U \to 1$ as $|\overrightarrow{x}| \to \infty$, required for finite energy. One could analyze in details the case of the spherically symmetry configuration, when the matrix field $U(\overrightarrow{x})$ has the form (the hedgehog):

$$U(\overrightarrow{x}) = \exp[i F(r) \overrightarrow{x}.\overrightarrow{\tau}]$$
 (1.1)

where $r = |\overrightarrow{x}|$ and $\overrightarrow{\tau}$ is the Pauli matrices. Here F(r) is a profile function with the boundary conditions $F(0) = n\pi$ (n is an integer) and $F(r) \to 0$ as $r \to \infty$. F(r) obeys a nonlinear differential equation which could be solve numerically.

An analytic form of the skyrmion profile function seems to be very useful for many purposes. Attemps were made to obtain a such analytic approximation: (i) by computing the holonomy of SU(2) Yang- Mills instantons in R^4 along lines paralell to the time -axis [4]; (ii) by identifying the skyrmion profile function with the sin-Gordon kink field (if we replace x by r) [5]. The approach (ii) is very attractive because of its simplicity. Moreover, unlike the instanton approach, the sin-Gordon kink has a fixed scale, so there are no abitrary scale parameters which have to be fixed by hand in order to minimize the energy. With the explicit expression of the kink-like profile function F(r), in Sec.2, the symmetry-breaking term of the Lagrangian

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is modified to obtain the hedgehog profile function for B=1 skyrmion which is in best coincidence with the kink-like profile function. Then we have calculated the energy of four different B=2 skyrmion configurations and lead to the result: the lowest value of the energy is obtained in the case of twisty configuration for the modified Lagrangian [6].

The results on numerical integration of the equation of motion for the time-dependent hedgehog indicate a dynamical chaostic character of fluctuations around the static soliton solution [7]. Such a study is interesting from the viewpoint of considering the Skyrme model as a nonlinear dynamical system. In Sec.3, we find the equation of motion for the time-dependent hedgehog of the considered model. The results of numerical integration of the obtained equation are presented. The fluctuation of the profile function $\delta F(x,t)$ as well as the time dependence of the amplitudes of different modes of the fluctuations are plotted.

The Skyrme's Lagrangian [1] is of fourth order in field derivatives. Various alternative models have been proposed which preserved the form of the original Lagrangian while extending it to higher orders [8-12]. The incorporation of the higher order terms, on the one hand, improves the fit of observables, and, on the other hand, gives a reasonable physical interpretation for stabilizing terms in Lagrangian. For example, one could introduce a sixth-order term [8-9]

$$\mathcal{L}^{(6)} = -\frac{\varepsilon_6^2}{2} B^{\mu} B_{\mu},\tag{1.2}$$

where B^{μ} is the baryon current

$$B^{\mu} = \frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2} Tr[(U^{+}\partial_{\nu}U)(U^{+}\partial_{\alpha}U)(U^{+}\partial_{\beta}U)]. \tag{1.3}$$

This term may be understood an representing effects of ϖ mesons, while the Skyrme's fourth-order term should be viewed as representing effects of ρ mesons. In Sec. 4, we consider the dynamical behavior of the model which the Skyrme Lagrangian added by the sixth-order term (1.2) and the modified symmetry-breaking term. The discussion of the results is given in Conclusion.

II. THE SIN-GORDON KINK FIELD AND THE CHOICE OF THE CHIRAL SYMMETRY BREAKING TERM

The Skyrme's Lagrangian density takes the form [2,3]

$$\mathcal{L} = \frac{F_{\pi}^2}{16} Tr \left(\partial_{\mu} U \partial^{\mu} U^+ \right) + \frac{1}{32e^2} Tr \left[(\partial_{\mu} U) U^+, (\partial_{\nu} U) U^+ \right]^2. \tag{2.1}$$

One considers also the Skyrme's Lagrangian with a pion mass term [3]

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_{SB},\tag{2.2}$$

$$\mathcal{L}_{SB} = \frac{1}{8} m_{\pi}^2 F_{\pi}^2 Tr \left(U + U^+ - 2 \right). \tag{2.3}$$

From (2.1), based on equations of motion, one found the nonlinear differential equation for the B=1 hedgehog profile function

$$\frac{xF'}{2} + \left[\frac{x^2}{4} + 2Sin^2F\right]F'' + Sin2F\left(F'\right)^2 - \frac{Sin\left(2F\right)}{4} - \frac{Sin^2FSin\left(2F\right)}{x^2} = 0, \ (2.4)$$

where $x = eF_{\pi}r$ is the dimensionless radial distance.

The kink-like function [5] has the form

$$F(x) = 4\arctan\left(e^{-x}\right),\tag{2.5}$$

which satisfies the same boundary conditions. The kink-like profile function (2.5) has an exponential decay for large x [5]. The same of an asymptotic behaviour is also obtained when the Skyrme Lagrangian (2.1) is added by a chiral symmetry-breaking term as (2.3) [13]. Accordingly, the right-hand side of (2.4) is not zero, but equals to

$$\frac{\beta^2}{4}x^2SinF,\tag{2.6}$$

where $\beta = \frac{m_{\pi}}{eF_{\pi}}$, $m_{\pi}=140$ MeV, e=4.84, $F_{\pi}=108$ MeV. However, if one substitutes the kink-like profile function (2.5) in the left-hand side of (2.4) and compare with (2.6) (in Fig.1a and Fig.1b), it is seen that they are different. In order to make the equality to be satisfied approximately, we must modify the pion mass term as following

$$3.5 \times 10^{-7} \frac{\beta^2}{4} x^2 SinF. \tag{2.7}$$

After modification the pion mass term as (2.7), we have plotted it in Fig.1c. We see that the approximate equality may be acceptable. So, one should be able to choice the symmetry-breaking term as follow [6]

$$\mathcal{L}_{SB}^{(mod.)} = \frac{\varepsilon}{8} m_{\pi}^2 F_{\pi}^2 Tr \left(U + U^+ - 2 \right), \qquad (2.8)$$

where $\varepsilon = 3.5 \times 10^{-7}$.

Now, we consider an ansazt has the form [13]

$$N = \{Cosk\phi Sin\vartheta, Sink\phi Sin\vartheta, Cos\vartheta\}, \tag{2.9}$$

with this ansazt the soliton mass is

$$M = M_2 + M_4, (2.10)$$

$$M_2 = \frac{\gamma}{4} \int_0^\infty dx x^2 \int_0^\pi d\vartheta Sin\vartheta \{ (F')^2 + [k^2 + 1] \frac{Sin^2 F}{x^2} \}, \tag{2.11}$$

$$M_{4} = \gamma \int_{0}^{\infty} dx x^{2} \int_{0}^{\pi} d\vartheta Sin\vartheta \{ \left[\frac{Sin^{2}\vartheta}{Sin^{2}\vartheta} k^{2} + 1 \right] (F')^{2} + \frac{Sin^{2}F}{x^{2}} k^{2} \} \frac{Sin^{2}F}{x^{2}}, \qquad (2.12)$$

where $\gamma = \frac{\pi F_{\pi}}{e}$, k is integer- k=1 corresponds to the case of the sherically symmetry hedgehod, $k \ge 2$ to the case of twisty skyrmion configuration. The variation of (2.10) in F(x) give the following equation

$$[x^{2} + 2a\sin^{2}F]F'' + 2xF' + [a(F')^{2} - \frac{a}{4} - 2b\frac{\sin^{2}F}{x^{2}}]\sin(2F) = 0,$$
 (2.13)

where

$$a = \int_0^{\pi} [k^2 + 1] \sin \vartheta d\vartheta, \qquad (2.14)$$

$$b = k^2 \int_0^{\pi} \sin \vartheta d\vartheta, \tag{2.15}$$

which for k = 2 we have a = 10 and b = 8.

From above-mentioned results and the formulas (2.11, 2.12), we consider the different cases of B=2 skyrmion configurations [6].

The first way of obtaining B=2 Skyrme field is to alter the boundary condition on the hedgehog profile function so that $F(0) = 2\pi$ [14]. One can generate an approximation to this Skyrme field by using the kink-like profile function

$$F(x) = 8\arctan\left(e^{-x}\right),\tag{2.16}$$

substitute it to (2.11) and (2.12) with k=1, we get

$$M_a = M_2 + M_4 \simeq 46.7\gamma.$$
 (2.17)

The second way of obtaining B=2 skyrme field is to leave the boundary condition on the profile unchanged but to have the skyrme field rotate twice as rapidly as the radial vector x under a change in the azimuthal angle around an axis [15]. From(2.1), (2.11) and (2.12) with k=2, we get

$$M_b = M_2 + M_4 \simeq 38\gamma.$$
 (2.18)

In the third case, based on the Lagrangian with \mathcal{L}_{SB} (2.3) and k=2, we obtained the equation of motion which is the equation (2.13) added by the term (2.6) in the right hand side. Solve this equation, we have numerical dat file. Substituting it to (2.11) and (2.12) we get

$$M_c = M_2 + M_4 \simeq 38.4\gamma.$$
 (2.19)

Analogously, in the last case, based on the Lagrangian with $\mathcal{L}_{SB}^{(mod.)}$ and k=2 we obtained the equation of motion which is the equation (2.13) added by the term (2.7) in the right hand side. We find numerically F(x) (dat file) and substitute it to (2.11) and (2.12) we get

$$M_d = M_2 + M_4 \simeq 27.5\gamma. \tag{2.20}$$

III. THE EQUATION FOR THE TIME -DEPENDENT HEDGEHOG AND THE TIME EVOLUTION OF THE SOLITON SOLUTION .

We consider the Skyrme Lagrangian (2.1) is added by a chiral symmetry-breaking term (2.8). It is convenient to parametrize the SU(2) matrix field U(x) by the pion field isovector $\overrightarrow{\pi}(x)$ [16,17]

$$U(x) = \frac{1 + i \overrightarrow{\eta}(x) \overrightarrow{\tau}}{1 - i \overrightarrow{\eta}(x) \overrightarrow{\tau}},$$
(3.1)

where

$$\overrightarrow{\eta}(x) = \frac{\overrightarrow{\pi}(x)}{F_{\pi}},\tag{3.2}$$

and $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices. In the parametrization (3.1) the expressions of the Cartan forms $(\partial_{\mu}U)U^+$ for the SU(2) group have been calculated [16,17], and on the base of these expressions we get the expressions of the Lagrangian (2.1) an (2.8) in the form

$$\mathcal{L} = \frac{F_{\pi}^{2}}{2} \frac{(\partial_{\mu} \overrightarrow{\eta} \partial^{\mu} \overrightarrow{\eta})}{(1 + \overrightarrow{\eta}^{2})^{2}} - \frac{4}{e^{2}} \frac{[\partial_{\mu} \overrightarrow{\eta} \times \partial^{\mu} \overrightarrow{\eta}]^{2}}{(1 + \overrightarrow{\eta}^{2})^{4}}, \tag{3.3}$$

$$\mathcal{L}_{SB}^{(mod.)} = -\frac{\varepsilon}{2} m_{\pi}^2 F_{\pi}^2 \frac{\overrightarrow{\eta}^2}{\left(1 + \overrightarrow{\eta}^2\right)}.$$
 (3.4)

The time-dependent hedgehog corresponds to the following ansatz

$$\overrightarrow{\eta}(r,t) = tan\left[\frac{F(r,t)}{2}\right]\frac{\overrightarrow{r}}{r},$$
 (3.5)

where F(r,t) is the profile function. Corresponding to this ansatz, \mathcal{L} and $\mathcal{L}_{SB}^{(mod.)}$ are given by

$$\mathcal{L} = \frac{F_{\pi}^2}{8} \{ (\dot{F})^2 - (F')^2 - \frac{2\sin^2 F}{r^2} \} + \frac{1}{2e^2} \frac{\sin^2 F}{r^2} \{ 2(\dot{F})^2 - 2(F')^2 - \frac{\sin^2 F}{r^2} \}, \quad (3.6)$$

$$\mathcal{L}_{SB}^{(mod.)} = \frac{\varepsilon m_{\pi}^2 F_{\pi}^2}{4} \left(\cos F - 1\right). \tag{3.7}$$

The Hamiltonian is given by

$$H = 4\pi \int r^2 dr \left\{ \frac{F_{\pi}^2}{8} \left\{ (\dot{F})^2 + (F')^2 + \frac{2\sin^2 F}{r^2} \right\} + \frac{1}{2e^2} \frac{\sin^2 F}{r^2} \left\{ 2(\dot{F})^2 + 2(F')^2 + \frac{\sin^2 F}{r^2} \right\} + \frac{\varepsilon m_{\pi}^2 F_{\pi}^2}{4} (1 - \cos F) \right\}.$$

$$(3.8)$$

The variational equation for the profile function is

$$\frac{xF'}{2} + \left[\frac{x^2}{4} + 2Sin^2F\right](F^{''} - \ddot{F}) + Sin2F(F^{'2} - \dot{F}^2) - \frac{Sin\left(2F\right)}{4} - \frac{Sin^2FSin\left(2F\right)}{x^2}$$

$$-\frac{\varepsilon\beta^2}{4}x^2SinF = 0, (3.9)$$

where $\beta=\frac{m_{\pi}}{eF_{\pi}}$, $m_{\pi}=140$ MeV, e=4.84, $F_{\pi}=108$ MeV [14] , and $x=eF_{\pi}r$, $\tau=eF_{\pi}t$ are the dimensionless distance and time. The primes and the dots mean the derivatives with respect to x and τ , respectively. Hereafter t is always understood as the dimensionless time.

It is convenient to write F(x,t) in the form

$$F(x,t) = F(x) + \delta F(x,t), \qquad (3.10)$$

where F(x) is the profile function of the static hedgehog. To find $\delta F(x,t)$ we use the following boundary conditions

$$\delta F(0,t) = \delta F(L,t) = 0. \tag{3.11}$$

This condition is automatically satisfied by the harmonic expansion

$$\delta F(x,t) = \sum_{j=1}^{N-1} A_j(t) \sin\left(\frac{j\pi x}{L}\right), \qquad (3.12)$$

where L is the size of the spatial volume, N is the number of the points of the discretized spatial variable x, $A_j(t)$ are the amplitudes of jth fluctuation modes. These amplitudes are obtained by inverting the series given in (3.12)

$$A_j(t) = \frac{2}{L} \int_0^L \delta F(x, t) \sin(\frac{j\pi x}{L}) dx.$$
 (3.13)

In our calculation we choose L=16, N=128 and the initial excitation mode is j =16. We denote $A \equiv A_{16}(0)$. We have studied the time evolution of the system for the pertubation parameter A = 0.1. The equation (3.9) should be reduced to a system coupled second order differential equations for the time variable t. We solve this system by using the Runge-Kutta procedure and obtain the solutions for $\delta F(x,t)$ and $A_i(t)$. We have plotted the fluctuation $\delta F(x,t)$ at t=0, 100, 200, 300 and 500 in Fig.2a to Fig.2e, respectively. We see that apart from large fluctuations near $x=0, |\delta F(x,t)| \sim 0.1$. This is understandable as the Skyrmion dynamics is dominated by the small x-region and the deviation from the Skyrmion is small. The dynamical behavior of the system is understood better by observing the time evolution of the amplitudes of various harmonic modes. In Fig.3a to Fig.3c we have plotted $A_i(t)$ for j=8, 16, 32, respectively. It is seen that the amplitude of the initial mode decreases gradually and it has some kind of periodicity in the variation, while amplitudes of other modes increase on the average. The time evolution of the mode of j=8 and j=16 has a periodic behavior when it was consider in a small interval of the time. It is to be expected that after a longer interval of time, all the modes would be of the same magnitude leading to "thermalization" and "spatio-temporal chaos".

IV. DYNAMICAL BEHAVIOR OF SKYRMION IN A GENERALIZED SKYRME MODEL .

We consider the Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}_{SB}^{(mod.)}, \tag{4.1}$$

where $\mathcal{L}^{(2)} + \mathcal{L}^{(4)}$ is the Lagrangian of the Skyrme model (2.1), $\mathcal{L}^{(6)}$ is given by (1.2), $\mathcal{L}_{SB}^{(mod.)}$ is given by (2.8). In the parametrization (3.1), the expressions of the Cartan forms $(\partial_{\mu}U)U^{+}$ for the SU(2) group have been calculated [16,17]. We get the Lagrangian (1.2) in the form

$$\mathcal{L}^{(6)} = \frac{4\varepsilon_6^2}{3\pi^4} \frac{1}{\left(1 + \overrightarrow{\eta}^2\right)^6} \left\{ (\partial \overrightarrow{\eta})^6 - 3(\partial \overrightarrow{\eta})^2 (\partial_\alpha \overrightarrow{\eta} \partial^\beta \overrightarrow{\eta}) (\partial_\beta \overrightarrow{\eta} \partial^\alpha \overrightarrow{\eta}) + \right.$$

$$2(\partial_{\nu}\overrightarrow{\eta}\partial^{\beta}\overrightarrow{\eta})(\partial_{\beta}\overrightarrow{\eta}\partial^{\alpha}\overrightarrow{\eta})(\partial_{\alpha}\overrightarrow{\eta}\partial^{\nu}\overrightarrow{\eta})\}. \tag{4.2}$$

By the same caculation in Sec. III, we have obtained the variational equation for the profile function

$$(\frac{x^{2}}{4}+2Sin^{2}F+\frac{\gamma}{4}\frac{Sin^{4}F}{x^{2}})(\ddot{F}-F^{"})+(Sin\left(2F\right)+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}})(\dot{F}^{2}-F'^{2})-\frac{1}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin\left(2F\right)}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Sin^{2}FSin^{2}}{x^{2}}+\frac{\gamma}{4}\frac{Si$$

$$(\frac{x}{2} - \frac{\gamma}{4} \frac{Sin^4 F}{x^3})F' + \frac{Sin(2F)}{4} + \frac{Sin^2 F Sin(2F)}{x^2} + -\frac{\varepsilon \beta^2}{4} x^2 SinF = 0, \tag{4.3}$$

where $x=eF_{\pi}r$, $\tau=eF_{\pi}t$ are the dimensionless distance and time. The primes and the dots mean the derivatives with respect to x and τ , respectively, $\gamma=\frac{F_{\pi}^2\varepsilon_6^2e^4}{\pi^4}$, $\beta=\frac{m_{\pi}}{eF_{\pi}}$. We choose $\varepsilon_6^2=5fm^2$, and t is understood as the dimensionless time.

Based on the expressions (3.10) to (3.13) and the choice of L=16, N=128, $A \equiv A_{16}(0) = 0.1$, we consider the dynamical behavior of the system with Lagragian (4.1) by observing the time evolution of the amplitudes of various harmonic modes. We have plotted the profile function of the static hedgehog and the fluctuation $\delta F(x,t)$ at t=0, 100, 200, 300 in Fig. 4a to Fig. 4d. It is a clear indication that $\delta F(x,t)$ has violent fluctuations for substantially longer intervalls in x.

The plots of $A_j(t)$ for the cases of j= 8, 16, 64, 127 (Fig. 5a to Fig. 5d) indicate that the fluctuation amplitudes in the generalized Skyrme model which Lagrangian added by the sixth-order term develop much faster than in the Skyrme model. For this model, the amplitudes of "spontaneous" fluctuations appear after $t \approx 150$. One can say that a self-exitation of soliton takes place after $t \approx 150$.

V. CONCLUSION

We have calculated the energy of four B=2 skyrmion configurations. The comparison of the values of the energy in four cases shows that in the case of modified Lagrangian one could obtain B=2 configuration with lowest energy. That is, this configuration is more close to the real energy minimum of B=2 skyrmion.

The B=1 hedgehog kink-like profile function (2.5) is a convenient analytic approximation to the numerical solution. But how can modify the Lagrangian to obtain the profile function (2.5), the modification made in this paper is one of the answers to the question.

By integrating the equation (3.1) for the time-dependent hedgehog we have obtained the information about the dynamical behavior of the sotiton in the model (3.3, 3.4). We have plotted $\delta F(x,t)$ at various moments and the development of amplitudes of fluctuation modes $A_j(t)$. The plots of $A_j(t)$ for the case A=0.1 indicate that the process of thermalization takes place sooner, and the fluctuation amplitudes in the considered model develop much faster than in the Skyrme model (see [7]). Besides, in [7], for A=0.1 the amplitude of the initial (32nd) mode has extremely regular periodic behavior while in our initial (16nd) mode the amplitudes decreases gradually on the average.

When higher order terms are included, the theory becomes much more nonlinear, so it is clearly that the phase space around the soliton solution in the generalized Skyrme model was shown to be more stochastic than in the original Skyrme model.

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